Technology Slides

Econ 360

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Learning Outcomes

- Determine whether or not a given level of output is feasible given a firm's inputs.
- Graphically and algebraically describe the set of feasible production plans of a firm.
- Determine algebraically and graphically a firm's returns to scale.
- Compare and contrast a firm's feasible production plans in the short run vs in the long run.
- Mathematically describe a firm's tradeoff between two different inputs.

Where We Are

- We have completely analyzed the consumer problem, and therefore derived the demand side of the market from the ground up.

- We want to analyze markets, which have both a demand side and a supply side.

- The supply side is based on firms, so we need to talk about the optimization problem the firm faces.

- Just like the consumer, the firm has an objective and a constraint.

- These set of slides will be focused on the constraint.

The Basic Idea

- Firms turn **inputs** into **output**.
  - We use y's to denote outputs. I.e. $\{y_1, y_2, ..., y_n\}$.
  - These outputs have prices $\{p_1, p_2, ..., p_n\}$.
  - We use x's to denote inputs. I.e. $\{x_1, x_2, ..., x_m\}$.
  - These inputs have prices $\{w_1, w_2, ..., w_m\}$.
- In this class, we will mostly focus on one output y.
- We will also generally focus on two main inputs, **L**abor, or workers, and **K**apital, or machines/buldings/land/any non-labor input.
  - Econ decided that K should be for capital since we use C for costs generally.
  - We will use w for the price of Labor and r for the price of K/Capital.

The Basic Idea–Simplified

- Our firm uses L and K to make a given amount of Y according to a **Production Function** $Y = f(K, L)$.
- Consider any combination of L, K, and Y denoted (L, K, Y).
  - We call this a **Production Plan**.
- A production plan is **feasible** if we can actually produce a given amount of Y with the K and L we have.
  - E.g. If $Y = K + L$ and our production plan is (L,K,Y)=(1,1,1000), the production plan is not feasible.
  - More generally, a production plan is feasible if $Y \leq f(L, K)$.
- The **Technology Set** is the collection of all feasible production plans.

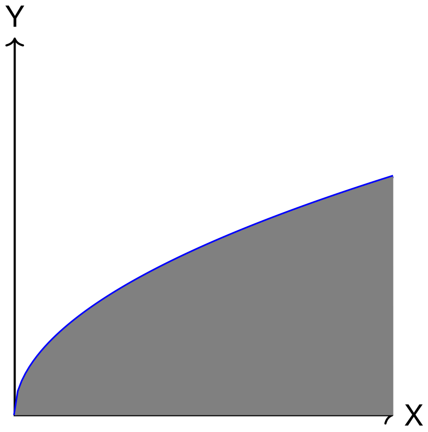
The Production Function/Technology Set

- Suppose we make it even simpler–1 output Y and 1 input X.
- As you increase X, how do you expect Y to change?
- Y should increase, but probably at a decreasing rate.
- We say the **Marginal Product** of X is increasing, but at a decreasing rate.
  - We denote the marginal product of X as MP_x.
- Or, we have **Diminishing Marginal Product**.
- In mathy terms,

$$\frac{\partial f(X)}{\partial X} > 0 \implies MP_x > 0$$

$$\frac{\partial MP_x}{\partial X} < 0.$$

Graphing a Production Function



- The Technology Set is the grey area and the blue line.
- The Production Function is just the blue line.

Production Functions with 2 Inputs

- Let's start with a **Cobb-Douglas Production Function.**
  - This function will look basically identical to Cobb-Douglas Utility functions from earlier in the semester.

$$Y = f(K, L) = K^{\alpha} L^{\beta}.$$

- We can calculate a marginal product for each of the two inputs.
- The marginal products of labor (MP_L) and capital (MP_K) are:

$$MP_L = \frac{\partial f(\cdot)}{\partial L} = K^{\alpha} \cdot \beta L^{\beta - 1}.$$

$$MP_K = \frac{\partial f(\cdot)}{\partial K} = \alpha K^{\alpha - 1} \cdot L^{\beta}.$$

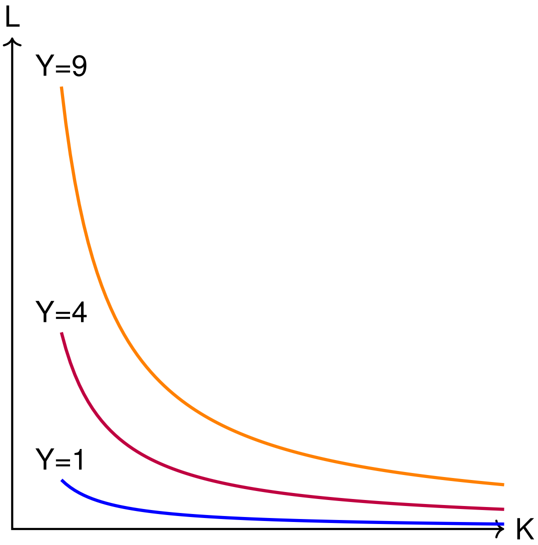
Tradeoff Between 2 Inputs

- These marginal products are useful because they tell us how one firm can trade between capital and labor while maintaining the same amount of Y, or output.
- Suppose a firm wants to produce 10 units.
- There are likely multiple combinations of capital and labor that result in an output level of 10.
- Because we are talking about "same quantity" we use the word **Isoquant** (or Isoquantity) to talk about all the combinations of the inputs that result in the same amount of output.
    - Exactly like how indifference curves are basically "isoutil" or "isoutility" curves.
- There are an infinite number of isoquant curves, one for each output level.

Graphing Isoquants

- We will graph isoquants the same way we graphed indifference curves.
  1. Pick an output level.
  2. Graph all input bundles that give us just barely that level of output.
     - "Just barely" is so we are on the production function rather than below the production function (blue line versus in the gray area from the graph earlier).
  3. Repeat steps and 1 and 2 to graph multiple isoquants.
- For example, suppose $Y = KL$.
- Let's graph the isoquants for Y=1, Y=4, and Y=9.

Graphing Isoquants–Example



The Slope of an Isoquant

- The idea here is the same as it was for an indifference curve.
- When the firm loses a worker, the firm loses the MP_L of that worker.
- In order to stay at the same level of production, it must add capital.
- The amount of capital the firm needs is based on MP_K.
- Specifically the amount is equal to $\frac{MP_L}{MP_K}$.
- We call this the **Marginal Technical Rate of Substitution** or MRTS.
- Sometimes you will also see it as just TRS.
- As you might imagine, this will be important when we talk about a firm's maximization problem in the next section.

Returns to Scale

- Earlier we talked about how it made sense that as we increase the amount of one input, holding the amount of the other inputs constant, output should increase at a decreasing rate.

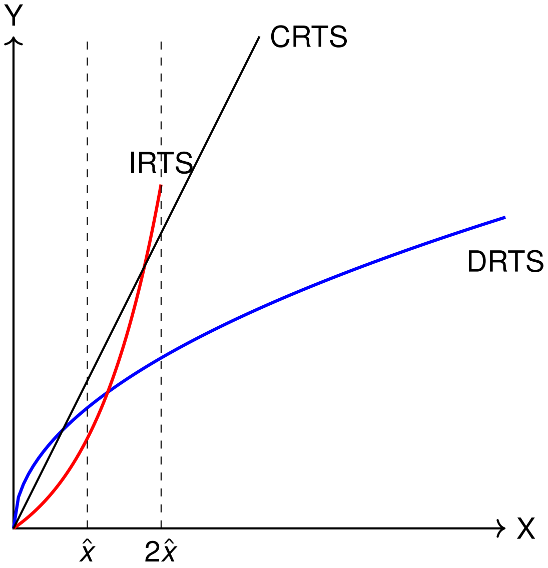
- That was an assumption we made and something that is not always true.

- **Question**: How can we determine if our assumption holds for a given production function or not?
  1. If the marginal products are diminishing for all inputs, that would tell us our assumption holds.
  2. An even easier way is to ask whether doubling our inputs results in a level of output that is less than double, double, or more than double.

Returns to Scale–Compare Doubling Inputs to Output

- Less than doubled output $\Longrightarrow$ **Decreasing Returns to Scale** or DRTS.
- Doubled output $\Longrightarrow$ **Constant Returns to Scale** or CRTS.
- More than doubled output $\Longrightarrow$ **Increasing Returns to Scale** or IRTS.
- Let's look at 3 production functions on the 1-input 1-output level to see what this looks like.

Returns to Scale-Graphed



Short vs Long Run

- In general, we assume it is easy to adjust the number of workers, or Labor.
- It is generally not possible in the short run, however, to change the amount of capital.
  - For example, a business cannot change which building they rent out or the number of heavy machinery they lease from another business, for example.
- So in the short run, we generally say K is fixed at $\bar{K}$ and the only thing we can change is L.
- Our production function becomes $Y = f(L, \bar{K})$.